Bessel Filters

Polynomials, Poles and Circuit Elements

This document contains tables of reference values for filter designers involved in the design of Bessel filters. The tables are new and represent the state of the art in machine calculation of filter parameters.

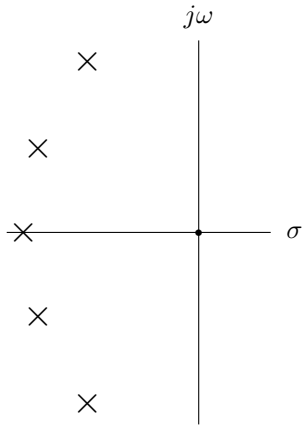
Polynomials, poles and element values are presented for normalized Bessel filters having unit delay at $\omega = 0$ as well as those having a 3dB cutoff frequency of 1 radian per second. The two sets of tables will be referred to as *delay normalized* and *frequency normalized*, respectively. In order to transform these values for a particular application it will be necessary to perform frequency and/or impedance scaling. Simple methods and formulas for such scaling can be found in any filter design text.

Element values are provided for implementing LC ladder solutions for each filter. The particular values given are for filters with equal source and load terminations, where R_S and R_L equal 1Ω. Because of the large number of possible source and termination resistances, the case of unequal terminations is better handled by a computer program than by tables. A suitable program for automatically designing filters of any order, frequency and terminations will be posted soon.

The table below provides accurate scale factors for converting Bessel filter parameters which have been normalized to unit delay at $\omega = 0$ to 3dB attenuation at 1 radian per second. These values were used in preparing the frequency normalized tables in this document.

Bessel Scale Factors

Order	Scale Factor
2	1.36165412871613
3	1.75567236868121
4	2.11391767490422
5	2.42741070215263
6	2.70339506120292
7	2.95172214703872
8	3.17961723751065
9	3.39169313891166
10	3.59098059456916



Pole locations for
5th order
Bessel filter

Delay Normalized Bessel Pole Locations

Order	σ	$j\omega$
1	−1.0000000000	0.0000000000
2	−1.5000000000	±0.8660254038
3	−1.8389073227	±1.7543809598
	−2.3221853546	0.0000000000
4	−2.1037893972	±2.6574180419
	−2.8962106028	±0.8672341289
5	−2.3246743032	±3.5710229203
	−3.3519563992	±1.7426614162
	−3.6467385953	0.0000000000
6	−2.5159322478	±4.4926729537
	−3.7357083563	±2.6262723114
	−4.2483593959	±0.8675096732
7	−2.6856768789	±5.4206941307
	−4.0701391636	±3.5171740477
	−4.7582905282	±1.7392860611
	−4.9717868585	0.0000000000
8	−2.8389839489	±6.3539112986
	−4.3682892172	±4.4144425005
	−5.2048407906	±2.6161751526
	−5.5878860433	±0.8676144454
9	−2.9792607982	±7.2914636883
	−4.6384398872	±5.3172716754
	−5.6044218195	±3.4981569179
	−6.1293679043	±1.7378483835
	−6.2970191817	0.0000000000
10	−3.1089162336	±8.2326994591
	−4.8862195669	±6.2249854825
	−5.9675283286	±4.3849471889
	−6.6152909655	±2.6115679208
	−6.9220449054	±0.8676651955

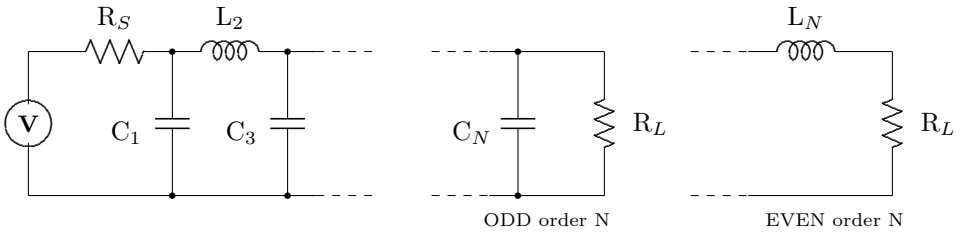
$$P_n(s) = \sum_{j=0}^{n} c_j \, s^j \quad (1)$$

$$= c_0 + c_1 \, s^1 + \cdots + c_n \, s^n \quad (2)$$

Delay Normalized Bessel Polynomial Coefficients

Order	Coefficient	Value
1	c_0	1.0000000000
	c_1	1.0000000000
2	c_0	3.0000000000
	c_1	3.0000000000
	c_2	1.0000000000
3	c_0	15.000000000
	c_1	15.000000000
	c_2	6.0000000000
	c_3	1.0000000000
4	c_0	105.00000000
	c_1	105.00000000
	c_2	45.000000000
	c_3	10.000000000
	c_4	1.0000000000
5	c_0	945.00000000
	c_1	945.00000000
	c_2	420.00000000
	c_3	105.00000000
	c_4	15.000000000
	c_5	1.0000000000
6	c_0	10395.000000
	c_1	10395.000000
	c_2	4725.0000000
	c_3	1260.0000000
	c_4	210.00000000
	c_5	21.000000000
	c_6	1.0000000000
7	c_0	135135.00000
	c_1	135135.00000
	c_2	62370.000000
	c_3	17325.000000
	c_4	3150.0000000
	c_5	378.00000000
	c_6	28.000000000
	c_7	1.0000000000

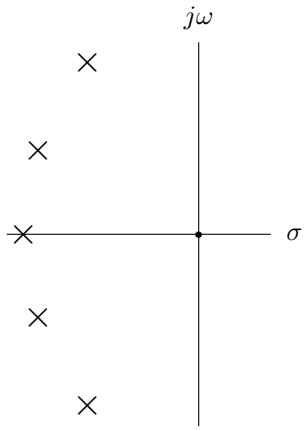
Order	Coefficient	Value
8	c_0	2027025.0000
	c_1	2027025.0000
	c_2	945945.00000
	c_3	270270.00000
	c_4	9450.0000000
	c_5	6930.0000000
	c_6	630.00000000
	c_7	36.000000000
	c_8	1.0000000000
9	c_0	34459425.000
	c_1	34459425.000
	c_2	16216200.000
	c_3	4729725.0000
	c_4	945945.00000
	c_5	135135.00000
	c_6	13860.000000
	c_7	990.00000000
	c_8	45.000000000
	c_9	1.0000000000
10	c_0	654729075.00
	c_1	654729075.00
	c_2	310134825.00
	c_3	91891800.000
	c_4	18918900.000
	c_5	2837835.0000
	c_6	315315.00000
	c_7	25740.000000
	c_8	1485.0000000
	c_9	55.000000000
	c_{10}	1.0000000000



Delay Normalized Bessel Filter Component Values
($R_S = R_L = 1\Omega$)
(Capacitance in Farads, Inductance in Henrys)

Order	Component	Value
1	C_1	2.0000000000
2	C_1	1.5773502692
	L_2	0.4226497308
3	C_1	1.2550242719
	L_2	0.5527864045
	C_3	0.1921893236
4	C_1	1.0598230345
	L_2	0.5116169398
	C_3	0.3181414385
	L_4	0.1104185872
5	C_1	0.9302987125
	L_2	0.4577030041
	C_3	0.3312217296
	L_4	0.2089636626
	C_5	0.0718128912
6	C_1	0.8376591607
	L_2	0.4115724743
	C_3	0.3158198657
	L_4	0.2364269328
	C_5	0.1480323182
	L_6	0.0504892483
7	C_1	0.7676537422
	L_2	0.3744134376
	C_3	0.2944134983
	L_4	0.2378303643
	C_5	0.1778259277
	L_6	0.1104061002
	C_7	0.0374569300

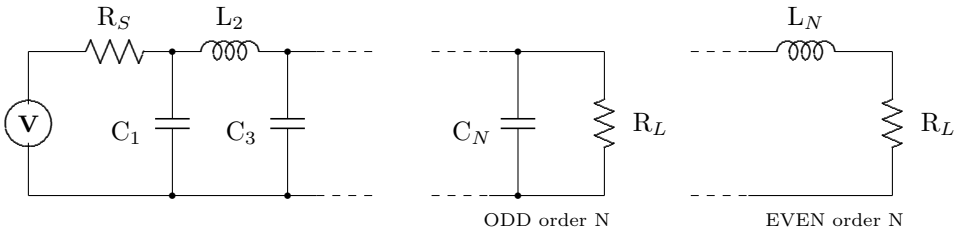
Order	Component	Value
8	C_1	0.7125408400
	L_2	0.3445569617
	C_3	0.2734607054
	L_4	0.2296681013
	C_5	0.1866805445
	L_6	0.1386714525
	C_7	0.0855168081
	L_8	0.0289045947
9	C_1	0.6677723576
	L_2	0.3202777482
	C_3	0.2547027138
	L_4	0.2183962295
	C_5	0.1859234132
	L_6	0.1505969673
	C_7	0.1111499572
	L_8	0.0681934544
	C_9	0.0229871832
10	C_1	0.6305035910
	L_2	0.3002229627
	C_3	0.2383951570
	L_4	0.2066336286
	C_5	0.1808239996
	L_6	0.1539454919
	C_7	0.1240448636
	L_8	0.0910809708
	C_9	0.0556979518
	L_{10}	0.0187012122



Pole locations for
5th order
Bessel filter

Frequency Normalized Bessel Pole Locations

Order	σ	$j\omega$
1	−1.0000000000	0.0000000000
2	−1.1016013306	±0.6360098248
3	−1.0474091610	±0.9992644363
	−1.3226757999	0.0000000000
4	−0.9952087644	±1.2571057395
	−1.3700678306	±0.4102497175
5	−0.9576765486	±1.4711243207
	−1.3808773259	±0.7179095876
	−1.5023162714	0.0000000000
6	−0.9306565229	±1.6618632689
	−1.3818580976	±0.9714718907
	−1.5714904036	±0.3208963742
7	−0.9098677806	±1.8364513530
	−1.3789032168	±1.1915667778
	−1.6120387662	±0.5892445069
	−1.6843681793	0.0000000000
8	−0.8928697188	±1.9983258436
	−1.3738412176	±1.3883565759
	−1.6369394181	±0.8227956251
	−1.7574084004	±0.2728675751
9	−0.8783992762	±2.1498005243
	−1.3675883098	±1.5677337122
	−1.6523964846	±1.0313895670
	−1.8071705350	±0.5123837306
	−1.8566005012	0.0000000000
10	−0.8657569017	±2.2926048310
	−1.3606922784	±1.7335057427
	−1.6618102414	±1.2211002186
	−1.8421962445	±0.7272575978
	−1.9276196914	±0.2416234710



Frequency Normalized Bessel Filter Component Values
($R_S = R_L = 1\Omega$)
(Capacitance in Farads, Inductance in Henrys)

Order	Component	Value
1	C_1	2.0000000000
2	C_1	2.1478055065
	L_2	0.5755027510
3	C_1	2.2034114362
	L_2	0.9705118162
	C_3	0.3374214850
4	C_1	2.2403786449
	L_2	1.0815160918
	C_3	0.6725248100
	L_4	0.2334158032
5	C_1	2.2582170510
	L_2	1.1110331705
	C_3	0.8040111711
	L_4	0.5072406309
	C_5	0.1743193807
6	C_1	2.2645236380
	L_2	1.1126429944
	C_3	0.8537858651
	L_4	0.6391554024
	C_5	0.4001898379
	L_6	0.1364923846
7	C_1	2.2659005520
	L_2	1.1051644360
	C_3	0.8690268432
	L_4	0.7020091536
	C_5	0.5248927291
	L_6	0.3258881312
	C_7	0.1105624499

Order	Component	Value
8	C_1	2.2656071373
	L_2	1.0955592547
	C_3	0.8695003726
	L_4	0.7302566539
	C_5	0.5935726770
	L_6	0.4409221408
	C_7	0.2719107171
	L_8	0.0919055476
9	C_1	2.2648789238
	L_2	1.0862838411
	C_3	0.8638734468
	L_4	0.7407329931
	C_5	0.6305951650
	L_6	0.5107787008
	C_7	0.3769865473
	L_8	0.2312912714
	C_9	0.0779654714
10	C_1	2.2641261600
	L_2	1.0780948329
	C_3	0.8560723826
	L_4	0.7420173503
	C_5	0.6493354737
	L_6	0.5528152742
	C_7	0.4454426979
	L_8	0.3270699985
	C_9	0.2000102641
	L_{10}	0.0671556901