Butterworth Filter Constants

1st Order

The magnitude response is

$$|H(jw)| = \frac{1}{\sqrt{1+\omega^2}}. \tag{1}$$

From the magnitude response, the minimum phase transfer function is

$$H(s) = \frac{1}{1+s}. \tag{2}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{1}{1+j\omega}. \tag{3}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}(\omega). \tag{4}$$

The group delay is

$$\tau(\omega) = \frac{1}{1+\omega^2}. \tag{5}$$

2nd Order

The magnitude response is

$$|H(jw)| = \frac{1}{\sqrt{1+\omega^4}}. \tag{6}$$

From the magnitude response, the minimum phase transfer function is

$$H(s) = \frac{1}{1+\sqrt{2}\,s+s^2}. \tag{7}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{1}{(1-\omega^2)+j(\sqrt{2}\,\omega)}. \tag{8}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{\sqrt{2}\,\omega}{1-\omega^2}\right). \tag{9}$$

The group delay is

$$\tau(\omega) = \frac{\sqrt{2}\,(1+\omega^2)}{1+\omega^4}. \tag{10}$$

3rd Order

The magnitude response is

$$|H(jw)| = \frac{1}{\sqrt{1+\omega^6}}. \tag{11}$$

From the magnitude response, the minimum phase transfer function is

$$H(s) = \frac{1}{1+2s+2s^2+s^3}. \tag{12}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{1}{(1-2\omega^2)+j(2\omega-\omega^3)}. \tag{13}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{2\omega-\omega^3}{1-2\omega^2}\right). \tag{14}$$

The group delay is

$$\tau(\omega) = \frac{2+\omega^2+2\omega^4}{1+\omega^6}. \tag{15}$$

4th Order

The magnitude response is

$$|H(jw)| = \frac{1}{\sqrt{1+\omega^8}}. \tag{16}$$

From the magnitude response, the minimum phase transfer function is

$$H(s) = \frac{1}{1+2.613126s+3.4142136s^2+2.613126s^3+s^4}. \tag{17}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{1}{(1 - 3.4142136\omega^2 + \omega^4) + j2.613126(\omega - \omega^3)}. \tag{18}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{2.613126(\omega - \omega^3)}{1 - 3.4142136\omega^2 + \omega^4}\right). \tag{19}$$

The group delay is

$$\tau(\omega) = \frac{2.613126 + 1.08239\omega^2 + 1.08239\omega^4 + 2.613126\omega^6}{1 + \omega^8}. \tag{20}$$

5th Order

The magnitude response is

$$|H(jw)| = \frac{1}{\sqrt{1 + \omega^{10}}}. \tag{21}$$

From the magnitude response, the minimum phase transfer function is

$$H(s) = \frac{1}{1 + 3.236s + 5.236s^2 + 5.236s^3 + 3.236s^4 + s^5}. \tag{22}$$

The corresponding amplitude response function in $j\omega$ is

$$H(j\omega) = \frac{1}{(1 - 5.236068\omega^2 + 3.236068\omega^4) + j(3.236068\omega - 5.236068\omega^3 + \omega^5)}. \tag{23}$$

The phase response is

$$\phi(\omega) = -\tan^{-1}\left(\frac{3.236068\omega - 5.236068\omega^3 + \omega^5}{1 - 5.236068\omega^2 + 3.236068\omega^4}\right) \tag{24}$$

The group delay is

$$\tau(\omega) = \frac{3.236068 + 1.236068\omega^2 + \omega^4 + 1.236068\omega^6 + 3.236068\omega^8}{1 + \omega^{10}}. \tag{25}$$