Problems and Solutions

in Mathematics, Physics and Applied Sciences

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Average Seek Distance for a Linear Actuator

Given a drive with T tracks, the maximum seek distance is $n = T - 1$. We wish to show that the average seek distance approaches 1/3 of the full distance between inner and outer tracks as the number of tracks approaches infinity. To begin, construct a transition matrix for the finite case designating the number of tracks to seek from each track to every other track. In this matrix, let the rows correspond to the

$$\begin{bmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 & n \\ 1 & 0 & 1 & & n-3 & n-2 & n-1 \\ 2 & 1 & & & & n-3 & n-2 \\ & \vdots & & \ddots & & \vdots & \\ n-2 & n-3 & & & & 1 & 2 \\ n-1 & n-2 & n-3 & & 1 & 0 & 1 \\ n & n-1 & n-2 & \cdots & 2 & 1 & 0 \end{bmatrix}$$

Figure 1: Track Seek Transition Matrix

present (start) track and the columns to the next (destination) track. Assuming that each seek is to a random track, the average number of tracks per seek (average stroke) is the sum of all entries divided by the number of entries.

$$A_T = \frac{1}{T^2} \sum_{ij} k_{ij} \tag{1}$$

The normalized distance is A_T/n for $T > 1$. To compute the sum in Eq.1, observe that the matrix is symmetrical along the principal diagonal, and the upper (lower) triangle represents the double sum of simple arithmetic series, so that

$$\frac{\text{average stroke}}{\text{full stroke}} = \frac{1}{nT^2}\sum_{ij} k_{ij} = \frac{1}{nT^2} 2\sum_{j=1}^{n}\sum_{k=1}^{j} k. \tag{2}$$

Now,

$$\sum_{ij} k_{ij} = 2\sum_{j=1}^{n}\sum_{k=1}^{j} k \tag{3}$$

$$= 2\sum_{j=1}^{n}\frac{j(j+1)}{2} = \sum_{j=1}^{n} j^2 + j \tag{4}$$

$$= (\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n) + (\frac{1}{2}n^2 + \frac{1}{2}n) \tag{5}$$

$$= \frac{1}{3}n^3 + n^2 + \frac{2}{3}n. \tag{6}$$

Subsituting in Eq. 2 and replacing n with $T-1$ we have

$$\frac{\text{average stroke}}{\text{full stroke}} = \frac{1}{T^3 - T^2}(\frac{1}{3}T^3 - \frac{1}{3}T) = \frac{1}{3} + \frac{1}{3T}.$$

In the limit, as the number of tracks increases to infinity

$$\frac{\text{average stroke}}{\text{full stroke}} = \frac{1}{3}.$$