

Optimum “L” Filters

Polynomials, Poles and Circuit Elements

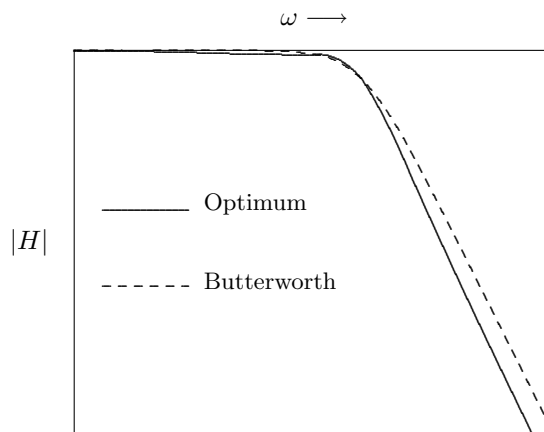
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In a 1958 paper Papoulis presented a new class of low pass filters with optimum properties.¹ These filters provide the maximum rolloff rate for a monotonic filter of given order. The price paid for this advantage is that the rolloff begins in the pass band and is uneven, although monotonic. But for some applications the aggressive noise suppression beyond the band edge may be attractive.

This document contains tables of reference values for filter designers involved in the design of Optimum or “L” filters. The tables are new and represent the state of the art in machine calculation of filter parameters.

Polynomials, poles and element values are presented for normalized “L” filters having a 3dB cutoff frequency of 1 radian per second. In order to transform these values for a particular application it will be necessary to perform frequency and/or impedance scaling. Simple methods and formulas for such scaling can be found in any filter design text.

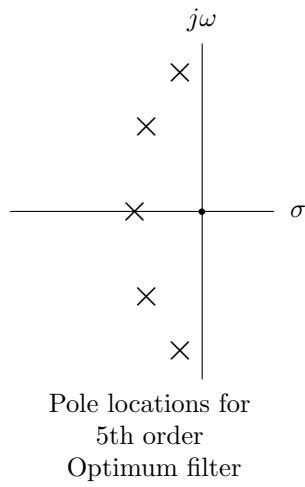
Element values are provided for implementing LC ladder solutions for each filter. The particular values given are for filters with equal source and load terminations, where R_S and R_L equal 1Ω . Because of the large number of possible source and termination resistances, the case of unequal terminations is better handled by a computer program than by tables.



Comparison of 3rd Order
Filter Response Curves

¹A. Papoulis, “Optimum Filters with Monotonic Response,” *Proc. IRE*, **46**, No. 3, March 1958, pp. 606-609.

Optimum Filter Pole Locations



Order	σ	$j\omega$
1	-1.0000000000	0.0000000000
2	-0.7071067812	± 0.7071067812
3	-0.3451856190 -0.6203318171	± 0.9008656355 0.0000000000
4	-0.2316887227 -0.5497434238	± 0.9455106639 ± 0.3585718162
5	-0.1535867376 -0.3881398518 -0.4680898756	± 0.9681464078 ± 0.5886323381 0.0000000000
6	-0.1151926790 -0.3089608853 -0.4389015496	± 0.9779222345 ± 0.6981674628 ± 0.2399813521
7	-0.0862085483 -0.2374347572 -0.3492317849 -0.3821033151	± 0.9843698067 ± 0.7783008922 ± 0.4289961167 0.0000000000
8	-0.0689421576 -0.1942758813 -0.3002840049 -0.3671763101	± 0.9879709681 ± 0.8247667245 ± 0.5410422454 ± 0.1808791995
9	-0.0550971566 -0.1572837690 -0.2485528957 -0.3093854331 -0.3256878224	± 0.9906603253 ± 0.8613428506 ± 0.6338196200 ± 0.3365432371 0.0000000000
10	-0.0459009826 -0.1325187825 -0.2141729915 -0.2774054135 -0.3172064580	± 0.9923831857 ± 0.8852617693 ± 0.6945377067 ± 0.4396461638 ± 0.1454302513

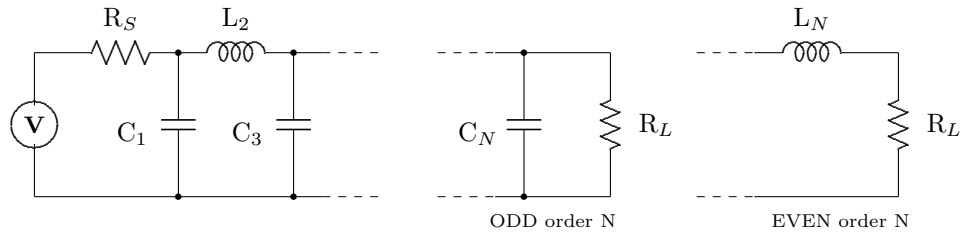
$$P_n(s) = \sum_{j=0}^n c_j s^j \quad (1)$$

$$= c_0 + c_1 s^1 + \dots + c_n s^n \quad (2)$$

Optimum Filter Polynomial Coefficients

Order	Coefficient	Value
1	c ₀	1.0000000000
	c ₁	1.0000000000
2	c ₀	1.0000000000
	c ₁	1.4142135624
	c ₂	1.0000000000
3	c ₀	0.5773502692
	c ₁	1.3589712494
	c ₂	1.3107030552
	c ₃	1.0000000000
4	c ₀	0.4082482905
	c ₁	1.2415698902
	c ₂	1.8879390659
	c ₃	1.5628642931
	c ₄	1.0000000000
5	c ₀	0.2236067977
	c ₁	0.8983414489
	c ₂	1.6927422745
	c ₃	2.2036429248
	c ₄	1.5515430544
	c ₅	1.0000000000
6	c ₀	0.1414213562
	c ₁	0.6796351916
	c ₂	1.6330772306
	c ₃	2.4334461480
	c ₄	2.6897282592
	c ₅	1.7261102278
	c ₆	1.0000000000
7	c ₀	0.0755928946
	c ₁	0.4379422692
	c ₂	1.2307978574
	c ₃	2.3322017528
	c ₄	2.9246254320
	c ₅	2.9927561302
	c ₆	1.7278634959
	c ₇	1.0000000000

Order	Coefficient	Value
8	c ₀	0.0451753951
	c ₁	0.2996706402
	c ₂	0.9939314565
	c ₃	2.1189254096
	c ₄	3.3477060909
	c ₅	3.7231573262
	c ₆	3.4466101114
	c ₇	1.8613567079
	c ₈	1.0000000000
9	c ₀	0.0238095238
	c ₁	0.1815729902
	c ₂	0.6804390044
	c ₃	1.7073835038
	c ₄	3.0119622871
	c ₅	4.2477427023
	c ₆	4.2489991402
	c ₇	3.7415869874
	c ₈	1.8663263313
	c ₉	1.0000000000
10	c ₀	0.0137464350
	c ₁	0.1168119035
	c ₂	0.4963112549
	c ₃	1.3754241530
	c ₄	2.8313316242
	c ₅	4.3545500296
	c ₆	5.5136006683
	c ₇	5.0886563697
	c ₈	4.1713681773
	c ₉	1.9744092560
	c ₁₀	1.0000000000



Normalized Optimum Filter Component Values

($R_S = R_L = 1\Omega$)

(Capacitance in Farads, Inductance in Henrys)

Order	Component	Value
1	C_1	2.0000000000
2	C_1	1.4142135624
	L_2	1.4142135624
3	C_1	2.1801141365
	L_2	1.3538072501
	C_3	1.1736931136
4	C_1	1.5643787973
	L_2	1.9585192860
	C_3	1.4768461419
	L_4	1.0826811970
5	C_1	1.9990424732
	L_2	1.5395135129
	C_3	2.0672625675
	L_4	1.4779915807
	C_5	0.9512000529
6	C_1	1.5763118669
	L_2	1.9040139855
	C_3	1.7442381095
	L_4	1.9857450780
	C_5	1.4851963494
	L_6	0.9159874632
7	C_1	1.8640432062
	L_2	1.5894515263
	C_3	2.1506430230
	L_4	1.7269910712
	C_5	1.9393934201
	L_6	1.4769890663
	C_7	0.8393520160

Order	Component	Value
8	C_1	1.5563512766
	L_2	1.8500853036
	C_3	1.8411428583
	L_4	2.0514702618
	C_5	1.7671680756
	L_6	1.9114767019
	C_7	1.4688302085
	L_8	0.8204601282
9	C_1	1.7644812270
	L_2	1.6134284249
	C_3	2.1584772888
	L_4	1.7816166010
	C_5	2.0662265226
	L_6	1.7754866342
	C_7	1.8674056310
	L_8	1.4555339287
	C_9	0.7694749200
10	C_1	1.5283251015
	L_2	1.8124064549
	C_3	1.8950693309
	L_4	2.0410837936
	C_5	1.8450909367
	L_6	2.0328534518
	C_7	1.7837880561
	L_8	1.8537513789
	C_9	1.4453410755
	L_{10}	0.7575192935