# Optimum "L" Filters

## Polynomials, Poles and Circuit Elements

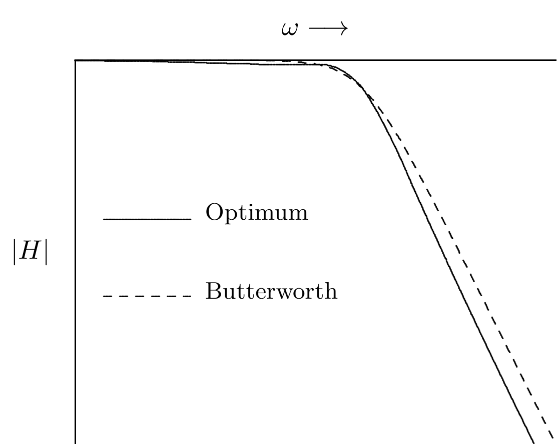


In a 1958 paper Papoulis presented a new class of low pass filters with optimum properties.[1] These filters provide the maximum rolloff rate for a monotonic filter of given order. The price paid for this advantage is that the rolloff begins in the pass band and is uneven, although monotonic. But for some applications the aggressive noise suppression beyond the band edge may be attractive.

This document contains tables of reference values for filter designers involved in the design of Optimum or "L" filters. The tables are new and represent the state of the art in machine calculation of filter parameters.

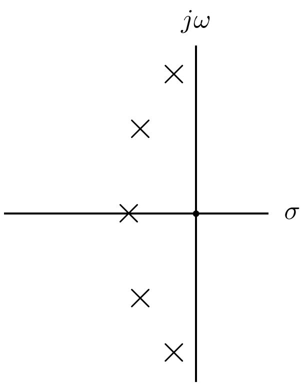
Polynomials, poles and element values are presented for normalized "L" filters having a 3dB cutoff frequency of 1 radian per second. In order to transform these values for a particular application it will be necessary to perform frequency and/or impedance scaling. Simple methods and formulas for such scaling can be found in any filter design text.

Element values are provided for implementing LC ladder solutions for each filter. The particular values given are for filters with equal source and load terminations, where $R_S$ and $R_L$ equal 1Ω. Because of the large number of possible source and termination resistances, the case of unequal terminations is better handled by a computer program than by tables.




Comparison of 3rd Order
Filter Response Curves

[1] A. Papoulis, "Optimum Filters with Monotonic Response," *Proc. IRE,* **46**, No. 3, March 1958, pp. 606-609.




Pole locations for
5th order
Optimum filter

**Optimum Filter Pole Locations**

| Order | $\sigma$ | $j\omega$ |
|---|---|---|
| 1 | −1.0000000000 | 0.0000000000 |
| 2 | −0.7071067812 | ±0.7071067812 |
| 3 | −0.3451856190 | ±0.9008656355 |
| | −0.6203318171 | 0.0000000000 |
| 4 | −0.2316887227 | ±0.9455106639 |
| | −0.5497434238 | ±0.3585718162 |
| 5 | −0.1535867376 | ±0.9681464078 |
| | −0.3881398518 | ±0.5886323381 |
| | −0.4680898756 | 0.0000000000 |
| 6 | −0.1151926790 | ±0.9779222345 |
| | −0.3089608853 | ±0.6981674628 |
| | −0.4389015496 | ±0.2399813521 |
| 7 | −0.0862085483 | ±0.9843698067 |
| | −0.2374397572 | ±0.7783008922 |
| | −0.3492317849 | ±0.4289961167 |
| | −0.3821033151 | 0.0000000000 |
| 8 | −0.0689421576 | ±0.9879709681 |
| | −0.1942758813 | ±0.8247667245 |
| | −0.3002840049 | ±0.5410422454 |
| | −0.3671763101 | ±0.1808791995 |
| 9 | −0.0550971566 | ±0.9906603253 |
| | −0.1572837690 | ±0.8613428506 |
| | −0.2485528957 | ±0.6338196200 |
| | −0.3093854331 | ±0.3365432371 |
| | −0.3256878224 | 0.0000000000 |
| 10 | −0.0459009826 | ±0.9923831857 |
| | −0.1325187825 | ±0.8852617693 |
| | −0.2141729915 | ±0.6945377067 |
| | −0.2774054135 | ±0.4396461638 |
| | −0.3172064580 | ±0.1454302513 |

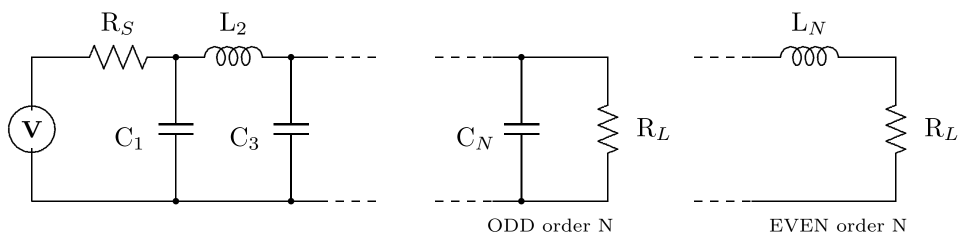
$$P_n(s) = \sum_{j=0}^{n} c_j\, s^j \quad (1)$$

$$= c_0 + c_1\, s^1 + \cdots + c_n\, s^n \quad (2)$$

**Optimum Filter Polynomial Coefficients**

| Order | Coefficient | Value |
|---|---|---|
| 1 | $c_0$ | 1.0000000000 |
| | $c_1$ | 1.0000000000 |
| 2 | $c_0$ | 1.0000000000 |
| | $c_1$ | 1.4142135624 |
| | $c_2$ | 1.0000000000 |
| 3 | $c_0$ | 0.5773502692 |
| | $c_1$ | 1.3589712494 |
| | $c_2$ | 1.3107030552 |
| | $c_3$ | 1.0000000000 |
| 4 | $c_0$ | 0.4082482905 |
| | $c_1$ | 1.2415698902 |
| | $c_2$ | 1.8879390659 |
| | $c_3$ | 1.5628642931 |
| | $c_4$ | 1.0000000000 |
| 5 | $c_0$ | 0.2236067977 |
| | $c_1$ | 0.8983414489 |
| | $c_2$ | 1.6927422745 |
| | $c_3$ | 2.2036429248 |
| | $c_4$ | 1.5515430544 |
| | $c_5$ | 1.0000000000 |
| 6 | $c_0$ | 0.1414213562 |
| | $c_1$ | 0.6796351916 |
| | $c_2$ | 1.6330772306 |
| | $c_3$ | 2.4334461480 |
| | $c_4$ | 2.6897282592 |
| | $c_5$ | 1.7261102278 |
| | $c_6$ | 1.0000000000 |
| 7 | $c_0$ | 0.0755928946 |
| | $c_1$ | 0.4379422692 |
| | $c_2$ | 1.2307978574 |
| | $c_3$ | 2.3322017528 |
| | $c_4$ | 2.9246254320 |
| | $c_5$ | 2.9927561302 |
| | $c_6$ | 1.7278634959 |
| | $c_7$ | 1.0000000000 |

| Order | Coefficient | Value |
|---|---|---|
| 8 | $c_0$ | 0.0451753951 |
| | $c_1$ | 0.2996706402 |
| | $c_2$ | 0.9939314565 |
| | $c_3$ | 2.1189254096 |
| | $c_4$ | 3.3477060909 |
| | $c_5$ | 3.7231573262 |
| | $c_6$ | 3.4466101114 |
| | $c_7$ | 1.8613567079 |
| | $c_8$ | 1.0000000000 |
| 9 | $c_0$ | 0.0238095238 |
| | $c_1$ | 0.1815729902 |
| | $c_2$ | 0.6804390044 |
| | $c_3$ | 1.7073835038 |
| | $c_4$ | 3.0119622871 |
| | $c_5$ | 4.2477427023 |
| | $c_6$ | 4.2489991402 |
| | $c_7$ | 3.7415869874 |
| | $c_8$ | 1.8663263313 |
| | $c_9$ | 1.0000000000 |
| 10 | $c_0$ | 0.0137464350 |
| | $c_1$ | 0.1168119035 |
| | $c_2$ | 0.4963112549 |
| | $c_3$ | 1.3754241530 |
| | $c_4$ | 2.8313316242 |
| | $c_5$ | 4.3545500296 |
| | $c_6$ | 5.5136006683 |
| | $c_7$ | 5.0886563697 |
| | $c_8$ | 4.1713681773 |
| | $c_9$ | 1.9744092560 |
| | $c_{10}$ | 1.0000000000 |




**Normalized Optimum Filter Component Values**
($R_S = R_L = 1\Omega$)
(Capacitance in Farads, Inductance in Henrys)

| Order | Component | Value |
|---|---|---|
| 1 | $C_1$ | 2.0000000000 |
| 2 | $C_1$ | 1.4142135624 |
| | $L_2$ | 1.4142135624 |
| 3 | $C_1$ | 2.1801141365 |
| | $L_2$ | 1.3538072501 |
| | $C_3$ | 1.1736931136 |
| 4 | $C_1$ | 1.5643787973 |
| | $L_2$ | 1.9585192860 |
| | $C_3$ | 1.4768461419 |
| | $L_4$ | 1.0826811970 |
| 5 | $C_1$ | 1.9990424732 |
| | $L_2$ | 1.5395135129 |
| | $C_3$ | 2.0672625675 |
| | $L_4$ | 1.4779915807 |
| | $C_5$ | 0.9512000529 |
| 6 | $C_1$ | 1.5763118669 |
| | $L_2$ | 1.9040139855 |
| | $C_3$ | 1.7442381095 |
| | $L_4$ | 1.9857450780 |
| | $C_5$ | 1.4851963494 |
| | $L_6$ | 0.9159874632 |
| 7 | $C_1$ | 1.8640432062 |
| | $L_2$ | 1.5894515263 |
| | $C_3$ | 2.1506430230 |
| | $L_4$ | 1.7269910712 |
| | $C_5$ | 1.9393934201 |
| | $L_6$ | 1.4769890663 |
| | $C_7$ | 0.8393520160 |

| Order | Component | Value |
|---|---|---|
| 8 | $C_1$ | 1.5563512766 |
| | $L_2$ | 1.8500853036 |
| | $C_3$ | 1.8411428583 |
| | $L_4$ | 2.0514702618 |
| | $C_5$ | 1.7671680756 |
| | $L_6$ | 1.9114767019 |
| | $C_7$ | 1.4688302085 |
| | $L_8$ | 0.8204601282 |
| 9 | $C_1$ | 1.7644812270 |
| | $L_2$ | 1.6134284249 |
| | $C_3$ | 2.1584772888 |
| | $L_4$ | 1.7816166010 |
| | $C_5$ | 2.0662265226 |
| | $L_6$ | 1.7754866342 |
| | $C_7$ | 1.8674056310 |
| | $L_8$ | 1.4555339287 |
| | $C_9$ | 0.7694749200 |
| 10 | $C_1$ | 1.5283251015 |
| | $L_2$ | 1.8124064549 |
| | $C_3$ | 1.8950693309 |
| | $L_4$ | 2.0410837936 |
| | $C_5$ | 1.8450909367 |
| | $L_6$ | 2.0328534518 |
| | $C_7$ | 1.7837880561 |
| | $L_8$ | 1.8537513789 |
| | $C_9$ | 1.4453410755 |
| | $L_{10}$ | 0.7575192935 |