Sundial Design

by C. Bond, 2010

Simple sundial

The simplest sundial possible, both in concept and implementation, consists of a slender rod and a hollow cylinder. The rod is rigidly placed along the axis of the cylinder and the assembly is then positioned so the rod is parallel to the earth's axis. In the northern hemisphere this effectively points the rod to the North star (Polaris).

It is convenient to mount the assembly securely in a sunny location and to remove the sunward side of the cylinder. With this arrangement the progress of the rod's shadow along the inner surface of the exposed cylinder serves as a clock.

As the earth rotates on its axis, it carries the sundial around a 360 degree arc. During the day, the shadow of the rod will move along the cylinder 15 degrees per hour ($15 \times 24 = 360$). The hours from approximately 6 am to 6 pm can be marked on the inner surface of the cylinder as lines parallel to the axis spaced 15 degrees apart. A plumb bob or other level is used to locate the 12 o'clock or noon line which is the line coincident with the shadow when the sun is at its peak.

Horizontal Sundial

The most familiar sundial is the horizontal sundial. This device reads the hour on a level, horizontal surface on which the gnomon is attached. In this case the gnomon could be a wire or rod slanted with respect to the surface and pointing North. Usually, however, for ridigidity, it consists of a triangular shaped sheet of solid material, which is firmly attached to the surface along one edge and perpendicular to the surface.

The longest elevated edge is in line with the axis of the earth. In other words, the reference edge form a sight line pointing North so that the angle of elevation of this line is the same as the latitude.

At high noon, for the horizontal sundial, the shadow of the gnomon on the surface appears as a straight line pointing North. This is the noon line.

To determine the shadow orientation for the other hour lines, both before and after noon, it is necessary to calculate their positions.

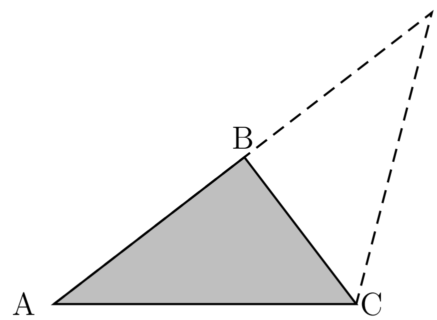

Figure 1: Gnomon Metrics for Horizontal Sundial

In Fig.(1) the triangle ABC is the gnomon for a horizontal sundial. It can be extended, as shown by the dashed lines, but only the shaded portion is needed for the following derivation. Angle $\angle BAC$ is set equal to the latitude at which the sundial will be located. Angle $\angle ABC$ is a right angle. One can think of the line BC as the radius of a circle perpendicular to AB centered at B, and corresponds to a cross-section through the cylinder described at the beginning of this article.

Hour lines are formed on the imaginary circle at B by dividing the circle in 15° intervals. The challenge is to determine where shadow will fall on the horizontal surface.

Derivation of Equations

Equations which describe the locations of the gnomon shadow at each hour are usually derived using spherical trigonometry. However, a simplified derivation can be done using only plane trigonometry.

In Fig.(2) which represents a three dimensional arrangement of triangles, the triangle ABC represents the gnomon described earlier. Angle $\angle BAC$ is equal to the latitude, and triangles ABC, BCD and ABD are right triangles, with triangle ABD in the plane of the horizontal sundial. Angle $\omega = CBD$ is

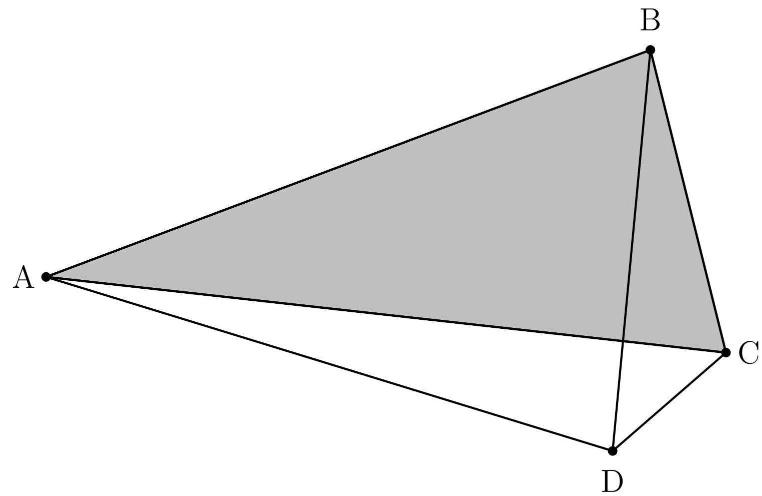

Figure 2: Geometry for Solving Sundial Equations

equal to a multiple of 15° and is in the plane perpendicular to line AB at B. Angle $\angle CAD$, is the angle from the mounting edge of the gnomon to the first shadow hour angle. This angle is to be calculated from the given values for the latitude and a multiple of 15°.

Now,

$$\frac{BC}{AC} = \sin(\lambda) \tag{1}$$

where λ is the latitude, $\angle BAC$.

$$\frac{CD}{BC} = \tan(\omega), \tag{2}$$

where ω is a multiple of 15°.

So,

$$\frac{BC}{AC}\frac{CD}{BC} = \frac{CD}{AC} = \tan(\phi) = \sin(\lambda)\tan(\omega) \tag{3}$$

where ϕ is the shadow hour angle for the desired hour before or after noon.

The solution equation is

$$\phi = \arctan(\sin(\lambda)\tan(\omega)). \tag{4}$$

By computing a table using $\pm 15°, \pm 30°, \pm 45°$... all possible hour lines can be found. These lines will be drawn on the horizontal surface from point A to point D.

My latitude, λ, is approximately $37.43°$. Hence a table for the hour lines of a horizontal sundial would be

Sun Time	Shadow Angle
7 pm	$-66.21°$
8 am	$-46.47°$
9 am	$-31.29°$
10 am	$-19.34°$
11 am	$-9.25°$
12 noon	$0.00°$
1 pm	$9.25°$
2 pm	$19.34°$
3 pm	$31.29°$
4 pm	$46.47°$
5 pm	$66.21°$

These angles are measured from the line where the gnomon joins the horizontal surface.

A Sundial Example

We are now in a position to design the layout for the horizontal surface of the sundial. Traditional designs place the gnomon on a circular surface which is mounted on some sort of pedestal. As previously stated, the gnomon is positioned so that it points North and the circular surface is level. The upper edge of the gnomon is aligned with the earth's axis.

For our design, we will take the line where the gnomon is mounted to the surface as the diameter of circular surface. It should be slightly smaller so there is room to place the hours near the edge of the circle.

The diagram in Fig.(3) will aid in calculating the locations of the hour shadow lines.

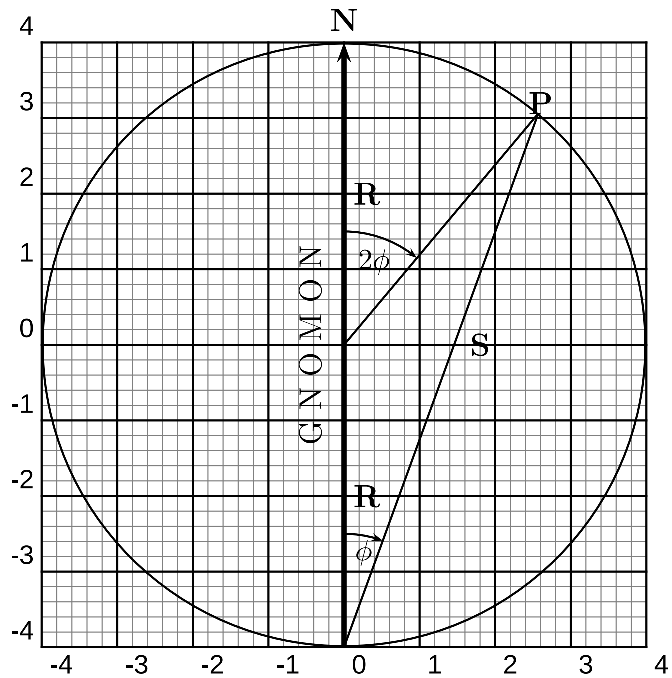

Figure 3: Geometry for Determining Hour Lines

"**S**" is the shadow line under consideration. "**R**" is the radius of the circle. ω is the angle for the hour as given in the previous table. "**N**" signifies North and point "**P**" is the end point of the shadow line under consideration.

Recall that the angle formed by the line from **P** to the center of the circle is 2ϕ. Hence, the location of point **P** can be found using a suitable protractor by striking an arc of 2ϕ from the noon or North point.

If the design is to be executed by computer, printed and placed on the circular surface, we need the coordinates of **P**. Assuming the circle is centered at the origin with radius **R** it is seen that the x coordinate of **P** is $\mathbf{R}\sin(2\phi)$. The y coordinate is $\mathbf{R}\cos(2\phi)$.

Universal Sundial

Although the sundial described in the previous paragraphs is only good for a specific latitude, it is possible to make a *universal* sundial by peoviding the

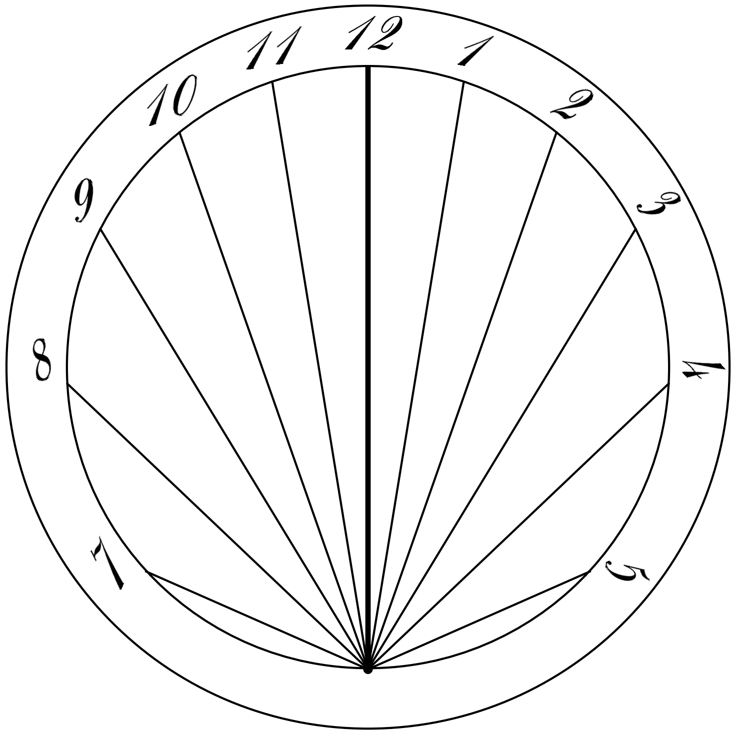

Figure 4: Sundial Layout

horizontal, circular surface with a suitable hinge. For example, if the north end of the surface is tilted up 5° the effective operating latitude is shifted 5° northhward.

Other improvements are possible. One is to allow the shadow hour lines and numbers to rotate so that the sun time can be set to account for daylight savings time or to read approximate local time.